Announcements

1) Math Club Talk

↳ Thursday,
CB 2062, on SVD

Recall: we were comparing algorithms for least squares

Best: QR Factorization via Householder

Algorithm: (11.2)

1) Compute the reduced QR decomposition of A
2) Compute $\hat{Q}^* b$
3) Solve $\hat{R} x = \hat{Q}^* b$ for x via back-substitution.

Next best: SVD

Algorithm: (11.3)

1) Compute reduced svd of A. algorithm?

2) Compute $\hat{U}^* b$

3) Solve $\hat{\Sigma} w = \hat{U}^* b$ via back-substitution

4) Set $x = V w$.

Worst: QR Factorization via Gram-Schmidt

Algorithm: (11.2)

Same as before, except use Gram-Schmidt instead of Householder to get the reduced QR decomposition.

But... the last algorithm isn't that accurate. We can fix it as follows:

Consider $B = \begin{bmatrix} A & b \end{bmatrix}$.

Revised Algorithm

reduced

1) Compute the QR decomposition of B via modified Gram - Schmidt.

2) Let $\hat{Q}b = R(1:n, n+1)$

3) Let $\hat{R} = R(1:n, 1:n)$

4) Solve for x via back substitution.

Theorem: (backwards stability)

The three algorithms for solving the least squares problem $Ax=b$ given by the SVD, QR decomposition using Householder reflections, and augmented QR decomposition via modified Gram-Schmidt are all backwards stable, i.e., for all three algorithms, the computed solution $\tilde{x}$ admits and $\tilde{A} \in \mathbb{C}^{m \times n}$

such that

$$\frac{\| A - \tilde{A} \|_2}{\| A \|_2} = O(\varepsilon_{machine})$$

and

$\| \tilde{A} \tilde{x} - b \|_2$ is

minimized.

Worse than Worst:

Normal Equations

Algorithm: 1) Form the normal equations

$$A^*Ax = A^*b$$

2) Solve using any method you like (MATLAB's "\" command)

Why is this so bad?

Look at our theory. We have a theorem that tells us we should expect to lose as many digits in our solution as the condition number.

Here, we're solving

$$A^* A x = A^* b,$$

and the condition number of $A^* A$ can be as large as $(\kappa(A))^2$!

So if we are accurate to 16 digits & $\kappa(A)$ costs us 8, we lose everything!

Theorem: (unstable) The previous algorithm for solving the least squares problem $Ax=b$ via the normal equations is not stable, and certainly not backwards stable.

If-Then Statements in Matlab

See the guides for further explanation, usually nested in a "for" loop and have the form

```
if (something)
    command
else (something)
    command
    ⋮
end
```